Voting by Committees

Barberà, Sonnenschein, and Zhou (1991, *Econometrica*)

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2026.07.09

What this paper does

- Society chooses a **subset of objects**, not just one winner.
- The paper asks: what do **truthful** voting rules look like?
- Main result: a full **characterization**.

Main message

$$\text{strategy-proofness} + \text{voter sovereignty} \iff \text{voting by committees.}$$

I will use one running example, then translate it into the paper's notation.

Running example: a club

Consider a club choosing new members.

Voters

$$N = \{1, 2, 3\}$$

Candidates / objects

$$K = \{a, b, c\}$$

The outcome is not a single candidate. It is any subset of candidates:

$$2^K = \{\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}.$$

This is the basic environment: subset choice.

Voting scheme

A voting scheme maps a profile of preferences into a chosen subset.

Formal object

$$f : P^n \longrightarrow 2^K.$$

- **Input**: each voter ranks subsets of K.
- **Output**: one chosen subset of K.

Think of f as the rule that turns individual rankings into a social outcome.

Separable preferences

The key domain restriction is **separability**.

Good objects

$$G(\succ) = \{x \in K : \{x\} \succ \varnothing\}.$$

Separability

For every $A \subseteq K$ and $x \notin A$,

$$A \cup \{x\} \succ A \quad \Longleftrightarrow \quad x \in G(\succ).$$

- If x is good, adding x is always good.
- If x is bad, adding x is always bad.
- There are no bundle interactions.

What separability rules out

A non-separable preference can look like this:

$$\{x\} \succ \varnothing, \qquad \{y\} \succ \varnothing, \qquad \text{but} \qquad \varnothing \succ \{x, y\}.$$

Interpretation

Each object alone is acceptable, but the bundle is bad.

- The value of x depends on whether y is also selected.
- This is exactly what separability excludes.

The positive theorem needs this restriction.

Best sets in the example

Under separability, the best set is just the set of good objects:

$$B(\succ_i) = G(\succ_i).$$

Voter	Likes	Best set
1	a, b	$B(\succ_1) = \{a, b\}$
2	a, c	$B(\succ_2) = \{a, c\}$
3	b, c	$B(\succ_3) = \{b, c\}$

In this domain, reporting the top set is the same as reporting good objects.

Committee for one object

For each object $x \in K$, define a committee

$$C_x = (N, \mathcal{W}_x).$$

Winning coalitions

$\mathcal{W}_x$ is the family of voter groups that can pass object x.

The only structural requirement is monotonicity:

$$M \in \mathcal{W}_x \text{ and } M \subseteq M' \implies M' \in \mathcal{W}_x.$$

If a group can pass x, any larger group can also pass x.

Example committees

Assign a different committee to each candidate.

- a: any 2 voters
- b: voter 1 + one more
- c: all 3 voters

$$\mathcal{W}_a = \{S \subseteq N : \#S \geq 2\}$$

$$\mathcal{W}_b = \{S \subseteq N : 1 \in S,\ \#S \geq 2\}$$

$$\mathcal{W}_c = \{\{1, 2, 3\}\}$$

Key idea

Different objects can have different power structures.

Computing the outcome

Given

$$B(\succ_1) = \{a, b\}, \qquad B(\succ_2) = \{a, c\}, \qquad B(\succ_3) = \{b, c\},$$

we check each object separately.

Object	Supporters	Committee test	Result
a	$\{1, 2\}$	$\{1, 2\} \in \mathcal{W}_a$	passes
b	$\{1, 3\}$	$\{1, 3\} \in \mathcal{W}_b$	passes
c	$\{2, 3\}$	$\{2, 3\} \notin \mathcal{W}_c$	fails

$$\boxed{f(\succ_1, \succ_2, \succ_3) = \{a, b\}}.$$

General definition

The example is summarized by one formula.

Voting by committees

$$x \in f(\succ_1, \ldots, \succ_n) \quad \Longleftrightarrow \quad \{i \in N : x \in B(\succ_i)\} \in \mathcal{W}_x.$$

- Left side: object x is selected.
- Right side: supporters of x are winning for x.
- The rule works one object at a time.

Why truth-telling?

Here, strategy-proofness means: truth-telling is a dominant strategy.

Take voter 1:

$$B(\succ_1) = \{a, b\}, \qquad c \text{ is bad.}$$

- If voter 1 adds bad object c, he may help c pass.
- If voter 1 hides good object a or b, he may make it fail.
- Under separability, neither lie helps.

Intuition

Truth-telling means reporting exactly the set of good objects.

Theorem 1

Theorem (Barberà, Sonnenschein, and Zhou (1991))

On the domain of separable preferences, a voting scheme f is ***strategy-proof*** *and satisfies* ***voter sovereignty*** *if and only if f is* ***voting by committees****.*

$$\boxed{\text{strategy-proof} + \text{voter sovereignty} \iff \text{voting by committees}}$$

Not just one example - a complete characterization.

What voter sovereignty adds

Voter sovereignty rules out degenerate rules.

Definition

For every $A \subseteq K$, there exists a profile $(\succ_1, \ldots, \succ_n)$ such that

$$f(\succ_1, \ldots, \succ_n) = A.$$

- No object is permanently banned.
- No object is automatically forced.
- Every subset can arise for some preference profile.

Proof idea only

The full proof is technical; the intuition is simple.

1. **Strategy-proofness** pushes the rule toward top-set dependence.
2. **Separability** lets us reason object by object.
3. For each object x, define $\mathcal{W}_x$ as the supporter groups that make x selected.
4. **Voter sovereignty** removes always-in / always-out cases.

Conclusion of the proof

The rule must be representable by object-specific winning coalitions.

Quota as a special case

If every object uses the same symmetric committee, we get quota voting.

Quota rule

$$x \in f(\succ) \quad \Longleftrightarrow \quad \#\{i \in N : x \in B(\succ_i)\} \geq Q.$$

- **Anonymity**: voter names do not matter.
- **Neutrality**: object names do not matter.
- With both conditions, voting by committees reduces to voting by quota.

Takeaway

- **Positive result**: truthful subset voting is possible.
- **Main result**: committees are not just an example; they are the whole class.
- **Price**: separability is a strong restriction.

Final message

Voting by committees is the complete class of strategy-proof and voter-sovereign rules on separable preferences.

Thank you.

Backup: non-separability and manipulation

Suppose

$$\{x\} \succ \varnothing, \qquad \{y\} \succ \varnothing, \qquad \varnothing \succ \{x, y\}.$$

Why this creates manipulation

If the voter reports both x and y as good, both may pass. But the bundle is worse than nothing.

- The voter may hide support for one object.
- This is why separability is not just technical.

Reference

Barberà, S., Sonnenschein, H., and Zhou, L. (1991).
Voting by Committees.
Econometrica, 59(3), 595–609.